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MAGNETO-HYDRODYNAMIC SHOCK STRUCTURE USING FRICTION

Cathleen S. Morawetz

January 12, 1959

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Contract No. AT(30-1)-1480

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Mathematics

MAGNETO-HYDRODYNAMIC SHOCK STRUCTURE USING FRICTION

by

Cathleen S. Morawetz

ABSTRACT

From the equations of motion for two charged fluids with scalar pressures it is shown that there exists a steady shock profile if there is friction between the two fluids proportional to the difference between their velocities.

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Introduction

Shocks as strict mathematical jump discontinuities occur in magnetohydrodynamics as well as in classical fluid mechanics if the motion is governed by conservation laws without friction terms. If friction terms are present, or introduced in the differential equations, the shock discontinuities are resolved into zones of rapid transition, and the behavior of the flow in these transition zones, the "shock profile" can be studied, much as ordinary boundary layer flow is studied, by greatly magnifying the width of the shock layer and considering the flow in the shock layer as a continuous solution of the corresponding differential equations which joins the states on the two sides of the layer. These states in turn are connected with each other by the classical shock relations, which are expressions of the conservation laws. Such an analysis of the shock profile is necessary to justify using discontinuous shocks as physically realistic models. In magnetohydrodynamics the problem is complicated by the fact that in many physically interesting cases the geometrical dimensions within which the flow takes place are small compared with the mean free path.

The present paper describes such a profile analysis under a trial assumption of a friction term which may be too much of an ad hoc idealization but allows rather complete discussion and analysis. If there is no friction then there is no shock but there are non-trivial steady state solutions. The theory is carried out for a two phase continuum fluid (positively-charged ions and negatively charged electrons) with scalar pressures in a one-dimensional steady situation, in which all quantities depend only on the space variable x and a shock zone connects two given states at $-\infty$ and $+\infty$. For results using a modified one phase fluid see [1], [2] and their bibliographies. For other results using a two-phase fluid see [7].

The method consists of reducing the problem to the solution of one ordinary nonlinear differential equation of first order with singularities.

The author gratefully acknowledges the advice and assistance of R. Courant. The computations were carried out on the Univac by H. Goertzel and the preliminary hand computations were made by Carole Siegel and Milton Vassell.

1. Equations of Motion

We shall consider the equations of motion expressing the usual conservation laws for the two fluids together with Maxwell's equations. We assume that the fluid is neutral which is an extremely accurate approximation to Maxwell's divergence equation for the electric field. This yields a system that is simple to handle and physically reasonable. We also assume that there is an interaction between the two fluids which may be interpreted as a friction or resistance. Since the flow depends on x only, these equations (see Grad [3] for their general form) in this special case reduce to:

$$(1) \qquad \frac{d}{dx}(\rho_r u_r^1) = 0 \qquad \text{(Conservation of Mass)}$$

$$(2) \qquad \frac{d}{dx}(\rho_r u_r^i u_r^1 + P_r^{i1}) = q_r(E^i + (u_r \times \mu H)^i) - \sum_{s=1,2} \alpha_{rs}(u_r^i - u_s^i)$$

(Conservation of Momentum)

$$(3) \qquad \frac{d}{dx}(\bar{e}_r u_r^1 + u_r^i P_r^{i1}) = E \cdot (q_r u_r) + \mathcal{E}_r \qquad \text{(Conservation of Energy)}$$

$$\left.\begin{aligned} &(4) \qquad \frac{d}{dx} H^1 = 0 \\ &(5) \qquad (\text{curl } H)^i = J^i = \sum_r q_r u_r^i \\ &(6) \qquad \frac{d}{dx} E^2 = \frac{d}{dx} E^3 = 0 \end{aligned}\right\} \qquad \text{(Maxwell's equations)}$$

where

$$(7) \qquad q_r = \gamma_r \rho_r .$$

Here, i = 1,2,3 denotes the component in the x,y, or z direction;

$r = 1,2$ denotes the ion and electron fluid respectively. ρ_r is density, u_r^i is velocity and P_r^{ij} is the pressure which we take to be scalar,

(8) $$P_r^{ij} = \delta_{ij} p_r \ .$$

J^i is current density, E^i (or E) is electric field, H^i (or H) is magnetic field, μ is the permeability constant, q_r is charge density and γ_r is the constant ratio of charge density to mass density $(\mathcal{E}_r/m_r)$, α_{rs} is the resistance coefficient with $\alpha_{12} = \alpha_{21} = \alpha > 0$. For the present we shall permit α to be a function of any of the dependent variables except the pressures. If it should depend on the pressures then it may only depend on $p_1 + p_2$. This approach can be modified for α depending separately on p_1 or p_2. $\bar{e}_r$ is total energy per unit volume $\bar{e}_r = e_r + \frac{1}{2}\rho_r u_r^2 = c_v p_r + \frac{1}{2} \rho_r u_r^2$.

The energy sources $\mathcal{E}_r$ satisfy the equation $\sum \mathcal{E}_r = 0$, that is, the total energy equation is unaffected by the internal resistance forces $\alpha_{rs}(u_r - u_s)$. To determine the electric field or the distribution of temperature one would need more knowledge of $\mathcal{E}_r$. This will be done for the case $\mathcal{E}_r = 0$.

We consider only the case where

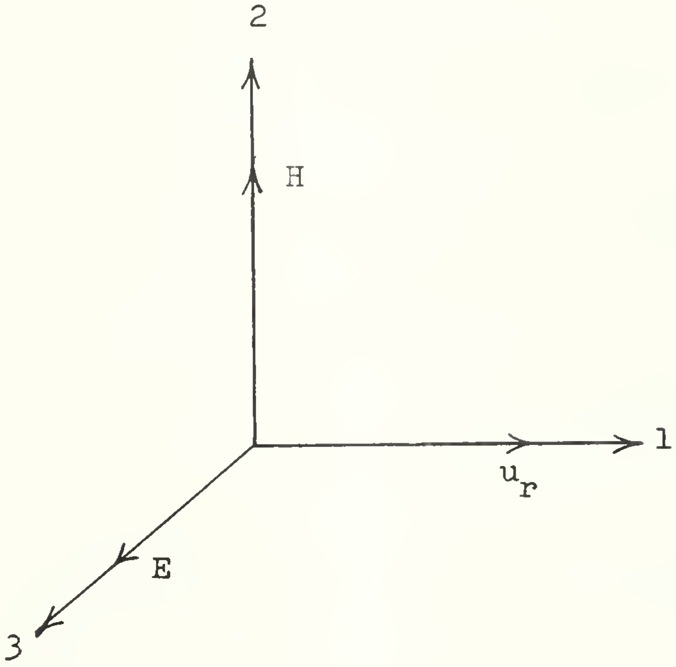
(9) $$H^1 = 0,$$

i.e., there is no component of magnetic field normal to the shock.

Finally, from the assumption that the total charge density is zero

(10) $$q_1 + q_2 = \sum_{r=1,2} \gamma_r \rho_r = 0 .$$

The boundary conditions at $-\infty$ are that the velocities and fields should be constant and we may choose coordinates so that $u_r^2 = u_r^3 = 0$, $H^3 = 0$, $E^2 = 0$.



2. Integration

Several of the equations can be integrated singly or in combinations. From (1), (5) and (10) we find

$$(11)\qquad \rho u^1 = M, \quad \rho = \rho_1 + \rho_2, \quad \rho_1 = \frac{-\gamma_2}{\gamma_1-\gamma_2}\,\rho, \quad \rho_2 = \frac{\gamma_1}{\gamma_1-\gamma_2}\,\rho ,$$

with

$$(12)\qquad u^1 = u_1^1 = u_2^1 ,$$

the speed of each fluid in the x-direction; M is constant. By adding the two momentum equations (2) in the y- and z-directions respectively and using (12), (8) and (10), we find

$$\frac{d}{dx}\left[\left(\sum \rho_1 u_r^i\right)u^1\right] = \sum_r q_r(u_r \times \mu H)^i = \left[\left(\sum q_r u_r\right) \times \mu H\right]^i, \quad (i=2,3)$$

By (9), (10) and (12) the right hand side vanishes, and hence, from the conditions at $-\infty$,

$$(13)\qquad \sum \rho_r u_r^i = 0 \qquad (i = 2,3).$$

Taking appropriate linear combinations of (2), we have by (8), (12), (13),

$$\frac{d}{dx}(J^i u^1) = -\gamma_1\gamma_2\rho E^i + \gamma_1\gamma_2\rho\mu u^1 \varepsilon_{ij} H^j + \alpha\,\frac{(\gamma_1-\gamma_2)^2}{\rho\gamma_1\gamma_2}\,J^i ,$$

where

$$\begin{aligned} \varepsilon_{ij}H^j &= H^3 && \text{for } i = 2, \\ &= -H^2 && \text{for } i = 3. \end{aligned}$$

That is, by (11), (5) and (4),

$$(14) \qquad E^2 = \frac{u^1}{\gamma_1\gamma_2 M}\frac{d}{dx}(u^1 \frac{d}{dx} H^3) + \mu u^1 H^3 - \frac{\alpha(\gamma_1-\gamma_2)^2}{\gamma_1^2\gamma_2^2} \frac{(u^1)^2}{M^2} \frac{dH^3}{dx}$$

$$(15) \qquad E^3 = - \frac{u^1}{\gamma_1\gamma_2 M}\frac{d}{dx}(u^1 \frac{d}{dx} H^2) - \mu u^1 H^2 + \frac{\alpha(\gamma_1-\gamma_2)^2}{\gamma_1^2\gamma_2^2} \frac{(u^1)^2}{M^2} \frac{dH^2}{dx}$$

where, by (6), E^2 and E^3 are constants. Without loss of generality we may take $E^2 = 0$. Then, from (14), we see that $H^3 = 0$ is a solution and satisfies the constant conditions at $-\infty$. Thus,

$$(16) \qquad J^2 \equiv E^2 \equiv H^3 \equiv 0.$$

We have reduced the differential equations of motion (1-10) to (15) where u^1 and H^2 are the dependent variables, and in addition there are two momentum equations in the x-direction (2) and two energy equations (3). We shall use only the total momentum and total energy equations to find the shock profile. The two remaining equations could be used to determine the charge separation field E^1 and the individual pressures.

The total momentum equation, found by adding the two equations of (2), integrates to

$$(17) \qquad Mu^1 + p + \frac{1}{2}\mu(H^2)^2 = P ,$$

where $p = p_1 + p_2$, and P is the constant of integration. The total energy equation (3) integrates to

$$(18) \qquad (\bar{e} + p)u^1 = E^3 H^2 + I$$

where I is the constant of integration.

Here

$$\begin{aligned}\bar{e} &= c_v(p_1+p_2) + \frac{1}{2}\sum \rho_r u_r \cdot u_r \\ &= c_v p + \frac{1}{2}(\rho_1+\rho_2)(u^1)^2 + \frac{1}{2}\sum_{i=2,3}\left(\rho_1(u_1^i)^2 + \rho_2(u_2^i)^2\right) \\ &= c_v p + \frac{1}{2}\,Mu^1 - \frac{u^1}{2M\gamma_1\gamma_2}\left(\frac{dH^2}{dx}\right)^2\end{aligned}$$

by (12), (11), (13) and (14). We may therefore write (18) as

$$(19) \qquad [(c_v+1)(P - \tfrac{1}{2}\mu(H^2)^2 - Mu^1) + \tfrac{1}{2}Mu^1 - \frac{u^1}{2M\gamma_1\gamma_2}(\frac{dH^2}{dx})^2]u^1 - E^3H^2 = I.$$

The problem has been reduced to solving (15) and (19) where u^1, H^2 are the dependent variables and M, E^3, I, P are constants of integration.* The total pressure p is found from (17), ρ_r, u_r^i, J^i from (5), (11), (12), (13).

---

*Note that $\alpha = \alpha(u_r^i, \rho_r, H^i, p_1+p_2) = \alpha(M, P, u^1, H^2, \frac{dH^2}{dx})$.

3. Conditions on Shock Solution

We shall look for a solution of (15) and (19) subject to the condition at $x = -\infty$ that the state is constant and show that the state to which one is led as $x \to +\infty$ is the corresponding state behind a shock provided the friction coefficient $\alpha > 0$.

The state at $x = -\infty$ satisfies

$$\rho = \rho_o, \quad p = p_o, \quad H^2 = H_o \text{ and } u^1 = u_o > \sqrt{\mu H_o^2 + (1+c_v^{-1})p_o/\rho_o}$$

(shock speed is greater than characteristic speed). Also $\frac{dH^2}{dx} = 0$ for $x = -\infty$ since $H^2 \to H_o$. From the standard theory of magneto-hydrodynamics (see, for example, Friedrichs and Kranzer [5]) we know that these constants determine the flow behind a shock. An appropriate profile theory must lead us to this flow at $x = +\infty$.

The constants of integration are found from (12), (15), (17), (19):

$$M = \rho_o u_o, \quad P = Mu_o + p_o + \tfrac{1}{2}\mu H_o^2 = \rho_o u_o^2 \left(1 + \frac{1+\beta_o}{2A^2}\right)$$

$$(20) \quad I = (c_v+1)p_o u_o + \tfrac{1}{2}Mu_o^2 - E^3 H_o = \tfrac{1}{2}\rho_o u_o^3 \left(1 + \frac{2+\beta_o(c_v+1)}{A^2}\right)$$

$$E^3 = -\mu u_o H_o = -\rho_o u_o^3 H_o^{-1} A^{-2}$$

where A is the Alfven Mach number, $\sqrt{\rho_o u_o^2/\mu H_o^2}$, and $\beta_o = 2p_o/\mu H_o^2$.

We introduce some convenient dimensionless variables:

$$\eta = \frac{u^1 dH^2/dx}{u_o H_o \sqrt{-\gamma_1\gamma_2\mu\rho_o}}, \qquad k = \frac{\alpha(\gamma_1-\gamma_2)^2}{(-\gamma_1\gamma_2 M)^{3/2}}\left(\frac{u_o}{\mu}\right)^{1/2},$$

(21)

$$\xi = \sqrt{-\gamma_1\gamma_2\mu\rho_o}\, x, \qquad H = \frac{H^2}{H_o}, \qquad u = \frac{u^1}{u_o}.$$

From (15) and (21), we obtain a pair of differential equations for η and H,

$$(22)\qquad \begin{aligned} u\,\frac{d\eta}{d\xi} &= -\,1 + uH - ku\eta \\ u\,\frac{dH}{d\xi} &= \eta \end{aligned}$$

Here u is a function of H and η given through the quadratic equation (19) which reduces by (20), since $c_v = \frac{1}{\gamma-1}$, to

$$(23)\qquad A^2(\gamma+1)u^2 - \gamma u[2A^2+1+\beta_o-H^2] + [A^2-\eta^2-2(H-1)](\gamma-1)+\gamma\beta_o = 0.$$

The appropriate root is determined by the initial conditions:

$$(24)\qquad u-1 = H-1 = \eta = 0, \qquad \xi = -\infty.$$

Since u is continuous we may switch to the other root only where the two roots are equal, that is, by (17), (20) and (23), where

$$(25)\qquad u^2 = \frac{\gamma p}{\rho u_o^2}.$$

The state behind the shock can be easily shown from the magnetohydrodynamic shock conditions to satisfy

$$(26)\qquad uH = 1, \qquad \eta = 0.$$

We note that (22) defines a direction field on a two-sheeted surface with coordinates η and H. On each surface u is defined by the appropriate root of (23) and the two sheets are joined where (25) holds. The direction field has singularities where (26) holds. Solutions for which u changes sign are physically meaningless. Likewise u is bounded since by (17), the pressure p would become negative for large u. The condition $p > 0$ reduces by (17), (20) and (21) to

$$(27) \qquad 2A^2(u-1) < 1-H^2 + \beta_o .$$

Finally, in general, no physically meaningful solution crosses the curve C where (25) holds. For, on C, the two roots of (24) are equal and hence at any point on C, $\frac{d\eta}{d\xi}$ and $\frac{dH}{d\xi}$ are the same on both sheets. It can be shown that, with a few exceptional cases, solution curves issuing from $\eta = 0$, $H = 1$ do not meet C tangentially and therefore to go from the first sheet to the second dH or $d\eta$ must change sign. Hence $d\xi$ changes sign and the solution is not single-valued in ξ (or x). Therefore

$$(27a) \qquad u^2 > \gamma p/\rho u_o^2 .$$

We therefore will never find a solution in which the velocity is less than gas sound speed. In particular the only possible shock profiles found by this method will be those for which the state behind the shock satisfies this condition. This limits the range of shock strength in terms of Alfven speed that we can consider. But infinitely strong changes in pressure are not excluded since the pressure ahead, or β_o can be as small as we like and the pressure behind is finite.

4. Nature of Singularities

The singularities of (22) occur where (26) holds. By (23) there are three such points. One point corresponds to the state ahead of the shock, one to a state behind the shock and one to a negative shock which we do not need to consider. The behavior near the singular points is given by

$$\eta = \mathrm{Re}\; a e^{\zeta\xi} + \ldots, \qquad H = 1 + \mathrm{Re}\; b e^{\zeta\xi} + \ldots \tag{28}$$

where ζ satisfies

$$\zeta = -\frac{k}{2} \pm \sqrt{\frac{k^2}{4} + u^{-2}(uH)_H} \tag{29}$$

evaluated at the singular point.

For the singular point corresponding to the state ahead of the shock, we find from (24),

$$\zeta = -\frac{k_o}{2} \pm \sqrt{\frac{k_o^2}{4} + \frac{2A^2 - 2 - \gamma\beta}{2A^2 - \gamma\beta}} \quad ;$$

k_o is the value of k ahead of the shock.

Since the shock speed relative to the gas is greater than the characteristic speed, we find $2A^2 - 2 - \gamma\beta > 0$. Hence ζ is real, and this point is a saddle point.

For $H = H_1$ say, $\eta = 0$, corresponding to the state behind the shock, we find using (20), (21), (23) and (24),

$$\zeta = -\frac{k_1}{2} \pm \sqrt{\frac{k_1^2}{4} + \frac{2A_1^2 - 2 - \gamma\beta_1}{(2A_1^2 - \gamma\beta_1)}}$$

where k_1 and u_1 are the values of k and u behind the shock, A_1 is the Alfven Mach number behind the shock,

$$A_1^2 = \frac{\rho(u^1)^2}{\mu(H^2)^2} = A^2u^3 ,$$

and $\beta_1 = \frac{2p}{\mu(H^2)^2} = \frac{p}{p_o}\beta_o$ behind the shock.

Since the speed of the shock is less than the characteristic speed behind it

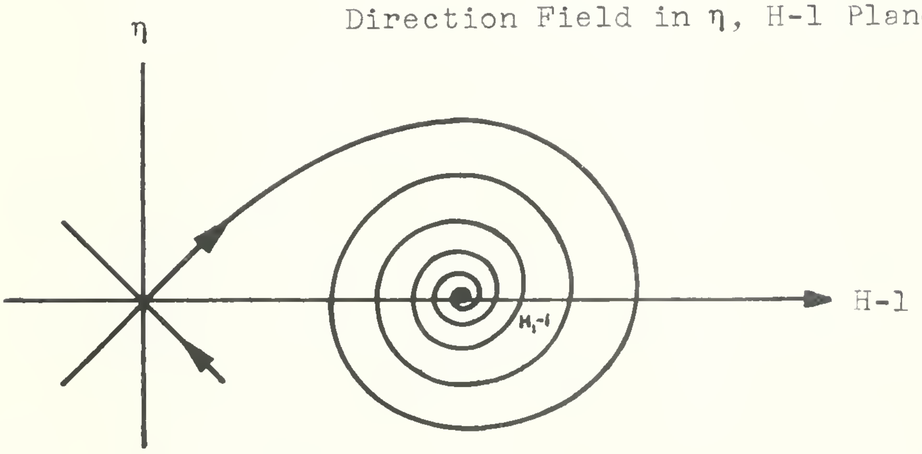
$$2A_1^2 < 2 + \gamma\beta_1 .$$

By (27a) only shocks with $u_1^2 > \gamma p/\rho u_o^2$ may be considered

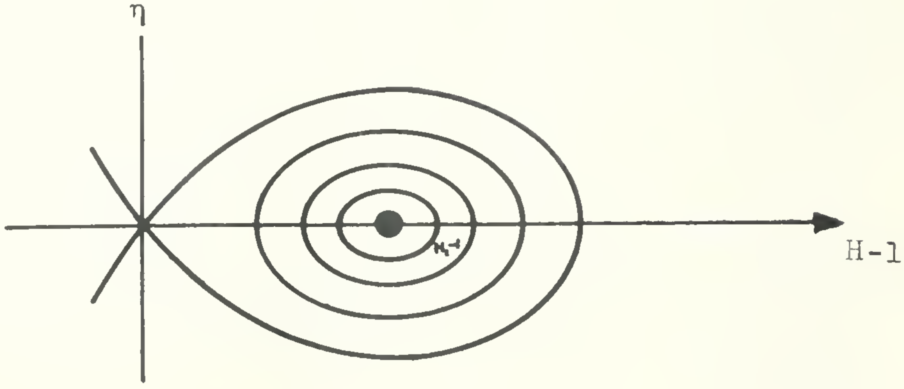
$$2A_1^2 > \gamma\beta_1 .$$

Hence the singular point corresponding to the state behind a shock is a focal point since $k_1 > 0$ and small as we shall see in Section 9.

Direction Field in η, H-1 Plane



$k_1 \neq 0$



$k_1 = 0$

5. Existence depending on $k_1 \neq 0$

It can be shown that if $u^2 > \gamma p/\rho u_0^2$ there will always be a continuous solution from one singular point to the other with $d\xi$ increasing if $k_1 \neq 0$.

In the important limiting case $k_1 = 0$ the focal point becomes a center and there is no solution that enters this point. We see from (22) and (23) that the solution in the (η, H)-plane is symmetric[*] about $\eta = 0$. The only solution issuing from the initial state is one which leads back to it (solitary wave) see Figures 1a,b. Thus there does not exist a shock solution. We note, however, the existence of steady state solutions which are periodic and, of course, do not satisfy the initial conditions.

When the shock solution exists it will look in the (η, H)-plane as in Figure 2a and, in the ξ,H-plane, the shock profile is as indicated in Figure 2b.

Finally, there is at most one curve leaving the saddle point (0) and entering the focal point (1). For, if two curves leave the initial point (0) with ξ increasing and enter the end point (1), the state behind the shock, then so does a curve leaving (0) with ξ decreasing. Therefore close to (1) these three curves circle (1) and enter the focal point. They intersect $\eta = 0$ with η increasing infinitely often and they never intersect each other. Therefore $d\eta/d\zeta$ changes sign infinitely often on $\eta = 0$ or by (22) $-1 + uH$ vanishes infinitely often on $\eta = 0$ which contradicts (24).

---

[*] Similar results (under slightly different assumptions) have been obtained by A. Banos, by Davis, Lüst, and Schluter, by H. Grad, and by M. Mittelman.

6. Wave Length and Shock Width

The wave length of the oscillations as $\xi \to \infty$ is given by

$$\lambda = 4\pi \left(\frac{-k_1^2}{4} - \frac{2A_1^2 - 2 - \gamma\beta_1}{2A_1^2 - \gamma\beta} \right)^{-1/2}$$

The wave length in the x-space by (21) is

$$\lambda(-\gamma_1\gamma_2\rho_o\mu)^{-1/2} .$$

Note that the characteristic length $X = (-\gamma_1\gamma_2\rho_o\mu)^{-1/2}$ is independent of the initial state except for the density.

We find next the order of magnitude of the damping coefficient using either Kolodner [6] or Spitzer [4]. By [4] equations 2-4, p. 18, with (2) we see that

$$P_{ie} = \alpha(u_1^i - u_2^i) = \alpha(v_e^i - v_t^i)$$

Comparing [2] equation (2-13) with (2-9) of this paper with $Z = 1$, $n_i = n_e$ and setting Spitzer's $n = N$

$$N = \left(\frac{c}{en_e}\right)^2 \frac{P_{ie}}{|v_i - v_e|} = \left(\frac{c}{en_i}\right)^2 \alpha$$

Here c is speed of light, e is charge in e.s.u., n_e = no. density, and N is given in e.m.u. by [4] page 86;

$$N = 1.29 \times 10^{13} \frac{\ell n \Lambda(T)}{T^{3/2}}$$

Then

$$k = \left(\frac{en_e}{c}\right)^2 \frac{NX}{\rho_o u_o} \left(\frac{m_i}{m_e}\right), \qquad X = \frac{\sqrt{m_i m_e}}{e\sqrt{\rho_o\mu}} ,$$

where e is given in e.s.u., μ in e.m.u.; or, since $m_e << m_i$,

$$X \sim \frac{I}{e} \frac{\sqrt{m_e}}{\sqrt{n_i \mu}} , \qquad k \sim \frac{e^2}{c^2} \frac{NX}{m_e u_o} .$$

These quantities are both independent of the prevailing magnetic field. Taking $n_i = 10^{15}$/c.c., $m_e = 9.035 \times 10^{28}$ gm. $e = 4.77 \times 10^{-10}$ abs. e.s.u. $\mu = 1.26 \times 10^{-6}$ henries/meter, $T = 10^8$ electron volts, we find $X = .017$ cm. and $k = 1.15 \times 10^3 u_o^{-1}$ sec/cm. with u_o in cm/sec.

Let bX by the distance from P_1 to P_2 where P_1 is the point with the lowest value of x for which $u-1 = .10(u_1-1)$ and P_2 be a point such that $|u-u_1| \leq .10u_o$ for $x \geq x(P_2)$. Then by numerical computation it is indicated, for the case with zero pressure ahead that with $k = 1/100$, b will be about 200. The details of the first cycles are in Figure 2b.

If b is about 200 the shock width is about 4 cm. which is a reasonable width for many Sherwood applications. Based roughly on the above examples, this corresponds to $u_o \sim 10^5$ cm/sec, that is, to a slow shock. In other words, only if the magnetic field and pressure ahead of the shock are small will the shock profile found here be significant. On the other hand as the speed of the shock becomes large the shock width bX rapidly becomes too large for Sherwood applications. This failure will occur in any theory base on collisions. It is necessary to study the existence of such fast shocks using the Boltzmann equations without collisions instead of a continuous theory.

These restrictions do not prevent us from getting large pressure ratios in the fluid by having sufficiently low fluid pressure compared to magnetic pressure (low β) ahead of the shock. In this sense strong shocks are admitted. The shocks have the character that their speed is not too different from the characteristic speed of the medium ahead, and in this sense the shocks considered are weak.

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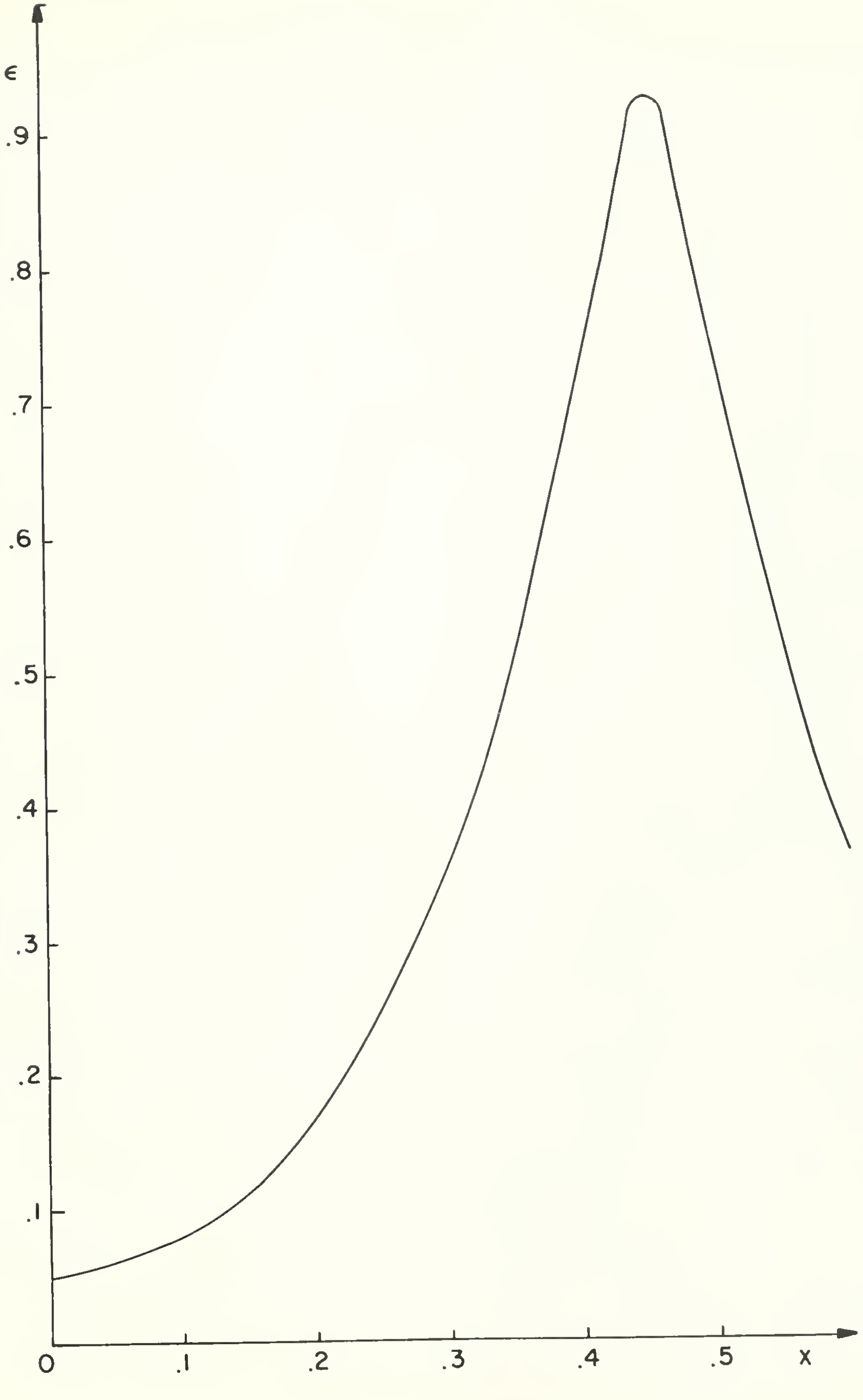
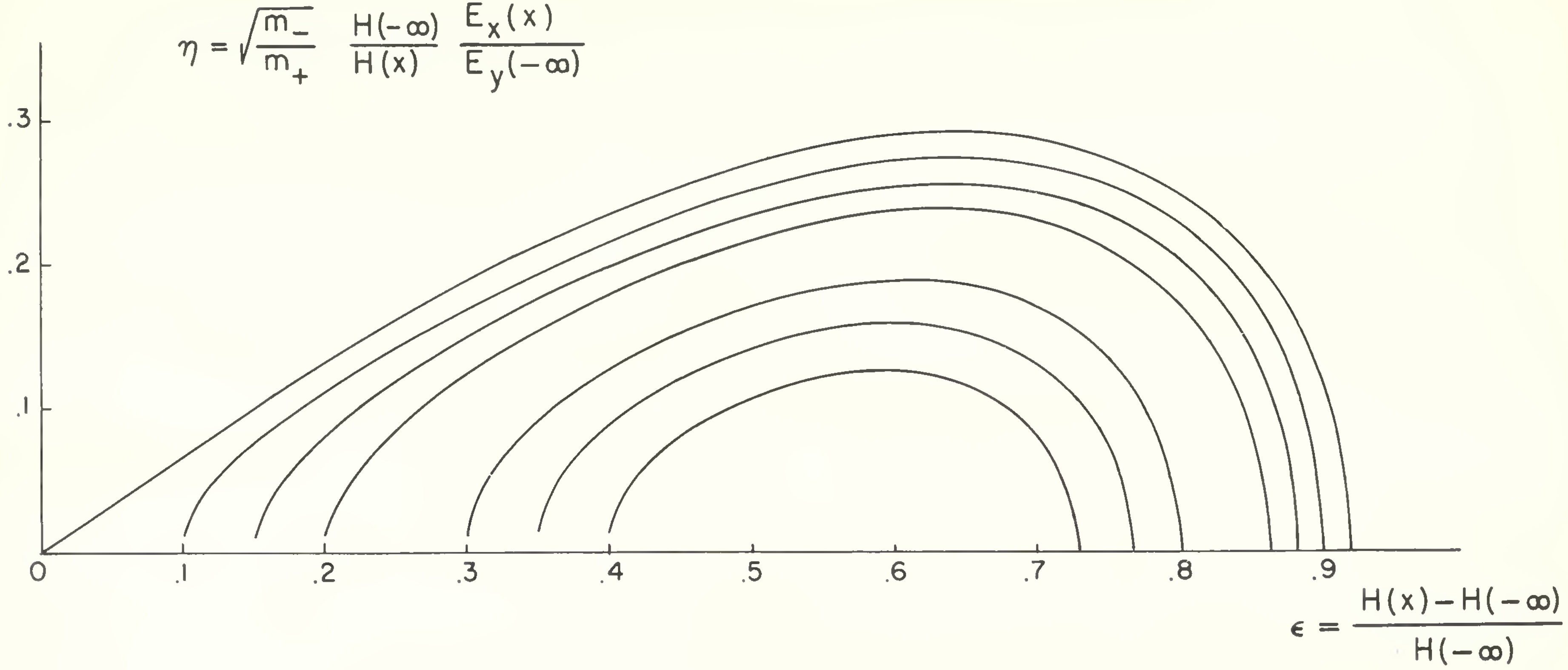


FIGURE 1b

$\eta = \sqrt{\frac{m_-}{m_+}} \; \frac{H(-\infty)}{H(x)} \; \frac{E_x(x)}{E_y(-\infty)}$
.3
.2
.1
0
.1
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.9
$\epsilon = \frac{H(x) - H(-\infty)}{H(-\infty)}$

FIGURE 1a

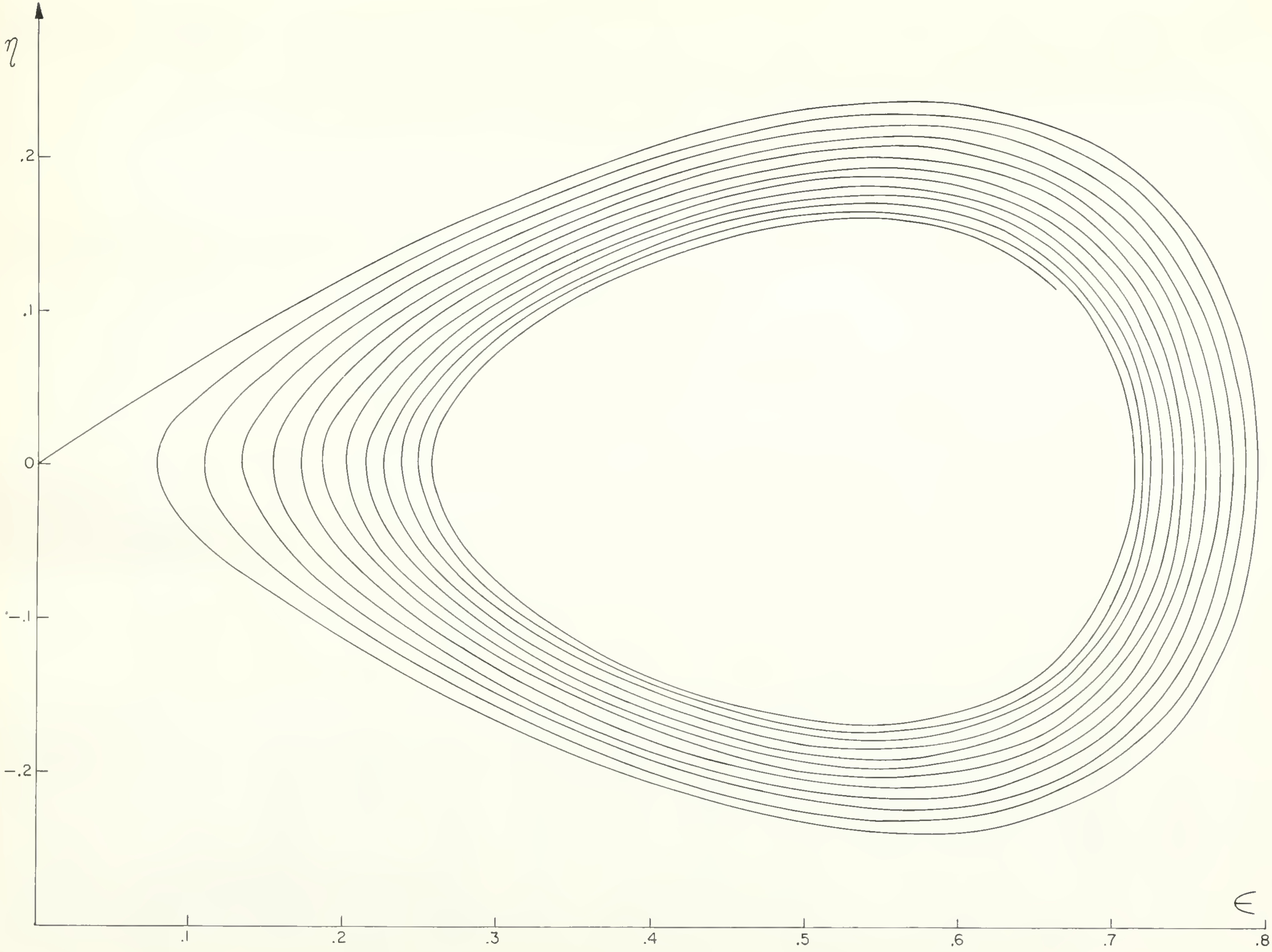
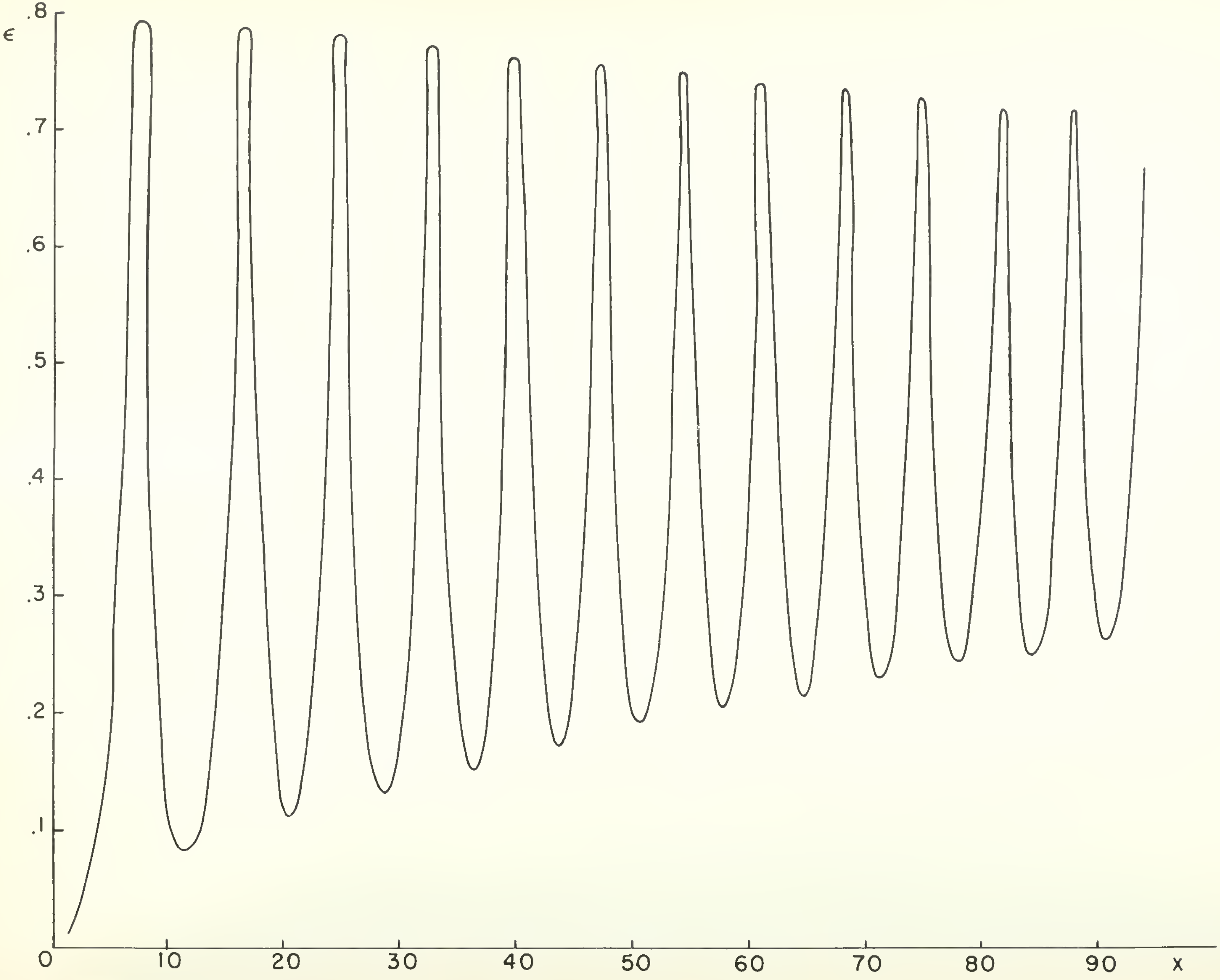


FIGURE 2a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